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Spectral Parameter Estimation by an Iterative Quadratic Maximum Likelihood Method

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An *i*terative *q*uadratic *m*aximum *l*ikelihood (IQML) method is applied to spectral parameter estimation of 1D NMR data. A careful comparison of the *l*inear *p*rediction (LP) method based on the singular value decomposition, the *t*otal *l*east *s*quares (TLS) method, and IQML has clearly demonstrated that IQML is superior to both the LP and TLS methods in terms of the accuracy and bias of the estimation. The superiority of the IQML method lies in the fact that constraints on the NMR signal can easily be incorporated into the iterative process. The iterative quadratic maximum likelihood method can be used to analyze NMR data directly or to provide a starting point for further data refinement. © 1998 Academic Press

Key Words: **iterative quadratic maximum likelihood method, IQML; linear prediction, LP; total least squares, TLS; data processing.**

INTRODUCTION

Efforts to overcome the intrinsic limitations of *f*ast *f*ourier *t*ransformation (FFT), such as low resolution and sidebands in truncated data and short time series due to limited observation time of the instrument or samples, have triggered developments of alternative spectral processing algorithms. For example, the maximum entropy (MEM) (*1–3*), linear prediction (LP) (*4–6*), total least squares (TLS) (*7*), and maximum likelihood (ML) (*8*) methods, among others, have been developed.

To quantitatively analyze NMR spectra, the linear prediction and total least-squares methods, based on singular value decomposition (*4*), which offer higher resolution than FFT in the case of truncation or short data series, can be used to estimate the amplitudes, phases, frequencies, and damping factors of resonances directly from an experimental FID without preprocessing. The LP and TLS methods, as parametric methods, have an advantage for the quantitative analysis of spectra in that these methods do not require complicated procedures to determine the intensities and frequencies of peaks and thereby allow easier estimation of the spectral parameters. However, one of the disadvantages of these approaches is that they are less straightforward to extend to multidimensional methods than nonparametrical approaches, such as the maximum entropy method (*3*).

The accuracy of the estimation of damping factors by the LP and TLS methods is especially sensitive to noise perturbations. This sensitivity in turn affects the accuracy of the estimation of amplitudes and phases. The present paper is concerned with an alternative parametric algorithm, the iterative quadratic maximum likelihood (IQML) method (*9, 10*) for spectral analysis. It has been demonstrated in a preliminary qualitative evaluation of the IQML method (*11*) that this method is much more accurate than the LP method. Here, a detailed comparison among the LP, TLS, and IQML methods shows that the IQML method is statistically superior to the LP and TLS methods in terms of the accuracy and bias of the estimated spectral parameter.

METHODS

In the application of the IQML method to NMR signal processing we are only interested in estimating the time domain signal which is a superposition of exponentially damped signals in noise. The model for the IQML method is the same as that of the linear prediction method and can be stated as follows.

Given a 1D FID signal of N equal-space sampled complex data points, $\{x_n | n = 0, 1, 2, \ldots, N-1\}$, and made up of M exponentially damped sinusoids embedded in random noise, the experimentally measurable data points x_n can be expressed as

$$x_n = \sum_{k=1}^{M} \{a_k \exp[j\phi_k]\} \exp[(\alpha_k + j2\pi f_k)n\Delta t] + \epsilon_n, \qquad n = 0, 1, 2, \ldots, N-1, \quad [1]$$

$$= \sum_{k=1}^{M} c_k z_k^n + \epsilon_n = x'_n + \epsilon_n,$$

where $j = \sqrt{-1}$, $c_k = a_k \exp[j\phi_k]$ with a_k and ϕ_k being the

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signal amplitude and phase, ϵ_n is the additive Gaussian noise, and $z_k = \exp[(\alpha_k + j2\pi f_k)\Delta t]$ with α_k and f_k being the damping factor and frequency, respectively. x'_n in Eq. [1] can be written into a matrix form

$$\mathbf{x}' = \mathbf{T}'\mathbf{c}', \qquad [2]$$

where $\mathbf{x}' = [x'_0, x'_1, \ldots, x'_{N-1}]^T$ denotes the vector of noiseless data. $\mathbf{c}' = [c_1, \ldots, c_M]^T$ is the vector of complex amplitudes, and the matrix $\mathbf{T}'$ is an $N \times M$ matrix with elements $\mathbf{T}'_{\mathbf{ij}} = z_j^i$. ^{T} indicates the transposition.

Under the white Gaussian noise assumption, the maximum likelihood (ML) method estimates of the vector $\mathbf{c}'$ and matrix $\mathbf{T}'$ are obtained by solving the following nonlinear least-squares problem $\min_{\mathbf{c}',\mathbf{T}'}\|\mathbf{x} - \mathbf{x}'\|$, where $\|\cdot\|$ represents the Euclidean norm, and the vector $\mathbf{x}$ is the given experimental data vector, $\mathbf{x} = [x_0, x_1, \ldots, x_{N-1}]^T$. For any $\mathbf{T}'$ the optimal estimate of $\mathbf{c}'$ is known to be $\mathbf{c}' = (\mathbf{T}'^+\mathbf{T}')^{-1}\mathbf{T}'^+\mathbf{x}$, and hence the least-squares error is

$$\mathbf{E} = \|\mathbf{e}\| = \mathbf{x}^+(\mathbf{I} - \mathbf{P_T})\mathbf{x}, \qquad [3]$$

where $^+$ denotes the Hermitian conjugate, and $\mathbf{P_T} = \mathbf{T}'(\mathbf{T}'^+\mathbf{T}')^{-1}\mathbf{T}'^+$ is the projection matrix of $\mathbf{T}'$. The nonlinear least-squares problem, Eq. [3], only involves one set of variables, $\mathbf{T}'$. According to the Linerar Prediction principle, there is a vector $\mathbf{b} = [b_0, b_1, \ldots, b_M]^T$ with $b_0 = 1$ such that the $(N - M) \times N$ Toeplitz matrix $\mathbf{B}$ defined as

$$\mathbf{B} = \begin{bmatrix} b_M & b_{M-1} & \ldots & b_1 & 1 & 0 & \ldots & \ldots & 0 \\ 0 & b_M & b_{M-1} & \ldots & b_1 & 1 & 0 & \ldots & 0 \\ . & . & . & \ldots & . & . & . & \ldots & . \\ . & . & . & \ldots & . & . & . & . & . \\ . & . & . & \ldots & . & . & . & . & . \\ 0 & . & b_M & b_{M-1} & . & . & . & b_1 & 1 \end{bmatrix} \qquad [4]$$

satisfies the equation $\mathbf{BT}' = 0$. Since $\mathbf{B}$ has rank $N - M$ and $\mathbf{T}'$ has rank M, this means that the project $\mathbf{P_B} = \mathbf{B}^+(\mathbf{BB}^+)^{-1}\mathbf{B}$ is the orthogonal complement to $\mathbf{P_T}$, *i.e.*, $\mathbf{P_B} = \mathbf{1} - \mathbf{P_T}$. Hence the least-squares error $\mathbf{E}$ can be expressed as

$$\mathbf{E} = \mathbf{x}^+\mathbf{P_B}\mathbf{x} = \mathbf{b}^+\mathbf{X}^+(\mathbf{BB}^+)^{-1}\mathbf{Xb}, \qquad [5]$$

where $\mathbf{X}$ is defined as

$$\mathbf{X} = \begin{bmatrix} x_M & x_{M-1} & \ldots & x_0 \\ x_{M+1} & x_M & \ldots & x_1 \\ . & . & \ldots & . \\ . & . & \ldots & . \\ . & . & \ldots & . \\ x_{N-1} & x_{N-1} & \ldots & x_{N-M-1} \end{bmatrix}, \qquad [6]$$

thereby satisfying $\mathbf{Bx} = \mathbf{X}\,\mathbf{b}$.

A maximum likelihood estimation algorithm, named the IQML method (*9, 10*), has been proposed to solve the above least-squares problem. It only requires the solution of a quadratic minimization problem at each step and converges in a small number of steps (normally less than 5). The proposed IQML algorithm (*11*), for the quantitative analysis of 1D NMR data, is composed of the following steps, (I) through (VII). Iteration of the method stops when the convergence criterion is met.

Step (I). Initialization,

set k = 0 and $\mathbf{b}_0 = [1, 1, \ldots, 1]$; or $\mathbf{b}_0$ = a constant vector.

Step (II). Compute

$$\mathbf{C}^{(k)} = \mathbf{X}^+(\mathbf{BB}^+)^{-1}\,\mathbf{X}, \qquad [7]$$

based on Eqs. [4] and [5].

Step (III). Solve the quadratic minimization problem

$$\min_{b_{(k+1)}\in\theta} \mathbf{b}^+_{(k+1)}\mathbf{C}^{(k)}\mathbf{b}_{(k+1)},$$

where θ is the constraint (see below) on the coefficients b_i, $i = 0, 1, \ldots, M$.

Step (IV). Increase k, $k = k + 1$.

Step (V). Check the convergence, $\|\mathbf{b}_{(k-1)} - \mathbf{b}_{(k)}\| < \epsilon$?

If yes, go to Step (VI).

If no, go to Step (II).

Step (VI). Find the frequencies, and damping factors from the roots of the characteristic polynomial formed by $\mathbf{b}_{(k)}$,

$$Z^M + b_1Z^{M-1} + b_2Z^{M-2} + \cdots + b_M = 0 \qquad [8]$$

and then the damping factors and frequencies are expressed as

$$\alpha_k = \frac{1}{\Delta t}\ln|Z_k|,$$

$$f_k = -\frac{1}{2\pi\Delta t}\tan^{-1}\frac{\mathrm{Im}(Z_k)}{\mathrm{Re}(Z_k)}.$$

Step (VII). Amplitudes and phases can be obtained by substituting the damping factors and frequencies obtained above into Eq. [1], and solving the corresponding least-squares problem again.

When $\mathbf{b}_0 = [1, 1, \ldots 1]$, the result obtained after the first iteration is the same as the LP method. The constraints that could be implemented for NMR applications in Step (III) are listed in the following: (a) constrain roots to be outside the unit circle for the backward LP arrangement to distinguish the signals from the noise, (b) Im($\mathbf{b}$) = 0 for real time series, (c) forward and backward LP constraints on $\mathbf{b}$ (*6*) which can be used for more accurate estimation, and (d) constrain roots to be

inside or on the unit circle for the stability of IQML. These constraints can be used individually or combined according to the application, thereby resulting in better estimates of spectral parameters.

RESULTS AND DISCUSSION

Similar parametric algorithms used in NMR applications, namely, the LP and TLS methods, employ a simpler linear least-squares approach (*4*, *5*, *7*) to estimate spectral parameters by using the fact that NMR data in general satisfy the model

$$x_n = -\sum_{L=1}^{M} b_L x_{n-L}, \qquad M \le n < N, \qquad [9]$$

where M and b_n are the LP order and the LP coefficients. This model is also implied in the IQML algorithm, Eqs. [4] and [5]. The above equation when applied to a time series can be written as

$$\mathbf{x} = -\mathbf{A}\mathbf{b},$$

where $\mathbf{A}$ is the $(N - M) \times M$ LP matrix, $\mathbf{b}$ is the forward LP vector, and the $\mathbf{x}$ is the observation vector. From the linear least-squares method, the LP matrix $\mathbf{A}$ can be decomposed by the SVD (singular value decomposition) method as

$$\mathbf{A} = \mathbf{U}\Lambda\mathbf{V}^{+}, \qquad [10]$$

where Λ is a diagonal matrix with its diagonal elements, λ_{ii}, the singular values of **A,** being positive numbers; $\mathbf{U}$ and $\mathbf{V}$ are the corresponding left and right singular matrices. $^{+}$ indicates conjugate transposition. The LP coefficient vector, **b,** can be found as

$$\mathbf{b} = \mathbf{V}\Lambda^{-1}\mathbf{U}^{+}\mathbf{x}. \qquad [11]$$

The spectral frequencies, damping factors, amplitudes and phases can then be calculated following Steps (VI) and (VII) of the IQML algorithm. It is known that the bias of SVD can be compensated for by subtracting the average of the singular values related to the noise from the singular values related to the signal (we abbreviated this method as the MLP or the modified LP method) (*5*) resulting in a more accurate estimation of the spectral parameters.

Furthermore, improvement can be achieved if noise is considered to perturb both the LP and observation matrices according to the TLS principle (*7*). The $(N - M) \times (M + 1)$ augmented matrix $\mathbf{A}' = [\mathbf{x}, \mathbf{A}]$ can be decomposed in the same manner as shown in Eq. [10]. The LP vector $\mathbf{b}'$ is shown to be

TABLE 1
The Theoretical Parameter Values of the Simulated Signal

k	f_k(Hz)	α_k(Hz)	$a_k \times 10^4$	ϕ_k(degrees)
1	3100.0	−18.0	40.0	70.0
2	2400.0	−7.0	18.0	80.0
3	1900.0	−9.0	30.0	30.0
4	1400.0	−15.0	24.0	150.0
5	1200.0	−13.0	20.0	90.0
6	−140.0	−12.0	15.0	180.0

$$\mathbf{b}' = -\sum_{i=p+1}^{L+1} \left[\frac{(v_k)_1^*}{\sum_{i=p+1}^{L+1} |(v_i)_1|^2} \right] \mathbf{v}'_k, \qquad [12]$$

where p is the effective rank of $\mathbf{A}'$; $\mathbf{v}'_k$ is the row partition matrix expressed as

$$\mathbf{v}_k = \begin{bmatrix} (v_k)_1^* \\ \mathbf{v}'_k \end{bmatrix} \qquad [13]$$

and * represents the complex conjugate. Following the same procedures as in Steps (VI) and (VII) of the IQML method, spectral parameters can be obtained. Since NMR data cannot, in general, be approximated by stationary signals, except when T_1 and T_2 of the signals are very long, we only adopt the forward or backward LP matrix, which contains no covariance matrix elements.

To statistically evaluate the performance of the LP, MLP, TLS, and IQML methods in estimating the spectral parameters of NMR signals, we quantitatively compared the accuracy of the LP, MLP, and TLS methods with that of the IQML method by applying them to the first 128 complex data points of simulated FID signals. This FID consisted of six components as listed in Table 1, and were sampled at 100 μs intervals, at a signal-to-noise ratio (SNR) of 20 dB. The SNR was defined as

$$\mathrm{SNR} = 10 \log \frac{a_{\max}^2}{\sigma^2}, \qquad [14]$$

where $a_{\max}$ and σ were the maximum amplitude value and the variance of added noise, respectively. For each estimation method, 50 separate trials were performed each using a different sequence to generate ϵ_n. The LP order, SNR, and the rank used were 26, 20 dB, and 6 correspondingly. The average of the estimated spectral parameters and their standard deviations are tabulated in Table 2. It is clearly shown in Table 2 that the IQML method yields much smaller Δ values (the sum of the difference of the theoretical values and their calculated values) and RMSD (root-mean-square deviation). This demonstrates

TABLE 2
Estimation of Spectral Parameters from LP, Modified LP, TLS, and IQML Methods with LP Order and True Rank Being 26 and 6, Respectively, and the Signal-to-Noise Ratio Is 20dB

k	f_k(Hz)	α_k(Hz)	$a_k \times 10^4$	ϕ_k(degrees)
LP				
1	3099.9 ± 0.9	−17.7 ± 1.0	39.9 ± 1.4	70.3 ± 2.0
2	2400.2 ± 1.3	−6.3 ± 1.2	17.5 ± 1.1	79.3 ± 3.2
3	1900.0 ± 0.7	−8.7 ± 0.8	29.8 ± 1.1	30.1 ± 1.9
4	1400.4 ± 1.4	−14.2 ± 1.2	24.3 ± 1.2	149.1 ± 3.4
5	1199.0 ± 1.4	−12.1 ± 1.6	19.4 ± 1.3	92.3 ± 3.1
6	−139.5 ± 2.0	−10.4 ± 2.1	14.2 ± 1.4	179.0 ± 4.7
Δ	2.3 ± 7.7	4.6 ± 7.9	2.5 ± 7.5	5.3 ± 18.3
MLP				
1	3099.9 ± 0.9	−17.7 ± 1.0	39.9 ± 1.4	70.3 ± 2.0
2	2400.2 ± 1.3	−6.3 ± 1.2	17.6 ± 1.1	79.3 ± 3.2
3	1900.0 ± 0.7	−8.7 ± 0.8	29.8 ± 1.1	30.1 ± 1.9
4	1400.4 ± 1.4	−14.3 ± 1.2	24.3 ± 1.2	149.2 ± 3.4
5	1199.0 ± 1.4	−12.2 ± 1.6	19.4 ± 1.3	92.2 ± 3.1
6	−139.5 ± 2.0	−10.5 ± 2.1	14.3 ± 1.4	179.0 ± 4.7
Δ	2.3 ± 7.7	4.3 ± 7.9	2.3 ± 7.5	5.1 ± 18.3
TLS				
1	3099.9 ± 0.9	−18.1 ± 1.0	40.2 ± 1.4	70.4 ± 2.0
2	2400.2 ± 1.3	−6.8 ± 1.2	18.0 ± 1.1	79.4 ± 3.1
3	1900.0 ± 0.7	−9.0 ± 0.8	30.1 ± 1.1	30.0 ± 1.9
4	1400.1 ± 1.4	−15.1 ± 1.3	25.1 ± 1.2	149.8 ± 3.4
5	1199.7 ± 1.4	−13.1 ± 1.6	20.1 ± 1.3	90.8 ± 3.1
6	−139.8 ± 2.0	−12.1 ± 2.1	15.1 ± 1.3	179.6 ± 4.7
Δ	0.9 ± 7.7	0.6 ± 8.0	1.6 ± 7.4	2.4 ± 18.2
IQML				
1	3099.7 ± 0.7	−18.0 ± 0.9	40.1 ± 1.0	70.5 ± 1.8
2	2400.1 ± 0.9	−6.9 ± 1.0	18.0 ± 1.0	79.6 ± 2.6
3	1900.0 ± 0.6	−9.0 ± 0.7	30.1 ± 0.9	30.0 ± 1.8
4	1400.1 ± 1.0	−15.2 ± 1.2	25.2 ± 1.0	149.7 ± 2.7
5	1199.7 ± 1.2	−13.1 ± 1.3	20.1 ± 1.3	90.6 ± 2.9
6	−140.0 ± 1.4	−12.0 ± 1.6	15.1 ± 1.1	180.1 ± 4.0
Δ	0.8 ± 5.8	0.4 ± 6.7	1.6 ± 6.3	1.9 ± 15.8

Note. $\Delta = \sum_{i=1}^{6}|A_i^T - A_i^M| \pm \sum_{i=1}^{6}|R_i|$, with A_i^T, A_i^M, and R_i being the theoretical values listed in Table 1, the corresponding mean values, and RMSD listed in Table 2.

that the IQML method is the least bias and the most accurate parametric method followed, in order, by the TLS, MLP, and LP methods.

It is well known that the performance of the LP, MLP, and TLS methods depends strongly on the LP order, rank, SNR, and the number of data points in the FID used in the calculation. In a more general qualitative comparison of these methods, we allowed the LP order, M, to vary from 6 to 50 and the SNR to vary from 10 to 20 dB. The effective rank p was set to be 6 since there are 6 resonances in the simulated signals and the first 128 complex data points of the simulated FIDs were used. It is advantageous to use the backward LP prediction scheme, as all the signal roots have to be outside the unit circle provided that the SNR is not too low, and the roots of the noise tend to fall inside the unit circle. This constraint was implemented in Step (III) of the IQML method. In these calculations, all the singular values after first p singular values were set to zero, and any root falling inside the unit circle was treated as an extraneous or noise signal. When the SNR of the signal is not high and the LP order is relatively small (LP order should be larger or equal to the "true" rank p), all methods will yield erroneous estimates or missing peaks. A systematic characterization of these methods was conducted to reveal the correlation between the LP order and errors of the estimation by varying the LP order and monitoring the estimation errors. In practice it was not straightforward to calculate estimation errors since in some trials classifying a peak as erroneous was difficult and some peaks could not be retrieved. To circumvent these problems the retrieval of a resonance was used to evaluate the methods as from the previous simulation we knew that the estimates of frequencies were normally quite accurate. For a peak to be considered retrieved, its frequency and amplitude had to satisfy $|f_t - f_e| < 100$ Hz and $|a_t - a_e| < 5 \times 10^4$, with the subscript t denoting the theoretical values in the simulated signals and e being the estimated values obtained from these methods. More relaxed or stringent criteria could be

TABLE 3
Number of Trials among Total 50 Trials in Which Estimations Did Not Miss Any Peak with the Rank Being 6

SNR (dB)	LP order											
	6	10	14	18	22	26	30	34	38	42	46	50
20												
LP	0	0	0	28	48	50	50	50	50	50	50	50
MLP	0	0	0	29	48	50	50	50	50	50	50	50
TLS	0	12	48	49	50	50	50	50	50	50	50	50
IQML	0	49	50	50	50	50	50	50	50	50	50	50
15												
LP	0	0	0	0	1	31	38	43	45	42	40	46
MLP	0	0	0	0	1	34	39	44	46	44	41	45
TLS	0	2	22	35	35	44	43	45	48	45	44	44
IQML	0	26	39	44	46	47	45	47	47	46	47	45
10												
LP	0	0	0	0	0	0	1	2	8	6	7	9
MLP	0	0	0	0	0	0	1	3	11	8	10	11
TLS	0	0	3	6	5	12	16	16	15	16	22	19
IQML	0	1	5	11	12	10	19	15	15	13	23	19

used; however, this would not affect the comparison of characteristics of the LP, MLP, TLS, and IQML methods in these conditions. The data listed in Table 3 clearly indicate that the IQML method is much less susceptible to noise perturbation. For example, the IQML method, with LP order and rank being 14 and 6 retrieved all 6 resonances in the simulated signals in 39 out of 50 trials (78%) at a 15-dB noise level, whereas, the only partially successful method, the TLS method retrieved these resonances in 22 of the trials (44%).

If the "true" rank of the experimental data is not known *a priori* and in order not to miss any peaks, we would normally choose a large number for the true rank p. We applied the LP, MLP, TLS, and IQML methods to the first 128 complex data points of a simulated FID containing 1024 complex data points with the LP order and the rank being 24. Figures 1a and 1b display the spectra obtained by FT of the simulated FID with and without additive noise. Figures 1c–1f show the spectra calculated from the LP, MLP, TLS, and IQML algorithms by averaging the 50 spectra calculated from the parameters estimated from each of the 50 simulated FIDs which contains a different random noise generated at a SNR of 10 dB. These spectra clearly indicate that the IQML algorithm is much more robust than the TLS and LP methods even when the rank is not known. From Table 2 and Fig. 1 of this study, the MLP method is only slightly more robust than the LP method and the TLS method is noticably better than both the MLP and LP methods.

It is important to point out that $\|\mathbf{b}_{(k+1)} - \mathbf{b}_{(k)}\| < \epsilon$ is not always applicable as the termination criterion. When the SNR is high, this criterion can be reached within 5 iterations in our test if $\epsilon = 10^{-4}$ was used. When the SNR is low, the algorithm may not converge by using this criterion. In the simulation reported here, we used 5 iterations in each case rather than this termination criterion. However, there is no guarantee that 5 iterations are proper for other cases. It is recommended to use 2 to 4 iterations in the general study.

Here, we demonstrate that this method is the most robust by processing an experimental 1D ^{13}C sucrose NMR spectrum, recorded on a Bruker ARX 300, when compared with the other paramatric methods described above. To make the spectral line widths more similar to those of ^{1}H or ^{31}P spectra, $Cr(acac)_3$ (0.01-*M* concentration) was added to the sample. In order to statistically compare the IQML method among the TLS and LP methods, we average the estimated IQML, TLS, and LP spectra calculated starting from the third, fourth, fifth, and sixth data points, with effectively different noise entries, with the LP order being the even numbers between 12 and 40. The first two data points were omitted because, due to switch off/on time of the receiver, these data were not measured accurately. The rank used was chosen to be 12 as there are 12 carbon peaks for this sucrose. The number of complex data points used for these calculation was 128 and 1024 complex data points were recorded with 20 scans. Figure 2A shows the 1D ^{13}C sucrose FFT spectrum. Figure 2B shows the 1D averaged ^{13}C sucrose LP reconstructed spectrum obtained according to the aforementioned conditions. Figures 2C and 2D show the 1D TLS and IQML reconstructed spectra in which the calculated phases were not used, since each calculation involved different starting points. Under these circumstances, the IQML method has retrieved all the resonances, whereas the TLS and LP spectra have either missing peaks or artifacts in amplitude and frequency due to inaccurate estimation of the frequencies and amplitudes of the resonance peaks. As the performance of the MLP method is very similar to the LP method (Fig. 1 and Table 3), it is not needed for this evaluation. Figure 2 clearly indicates that the IQML method

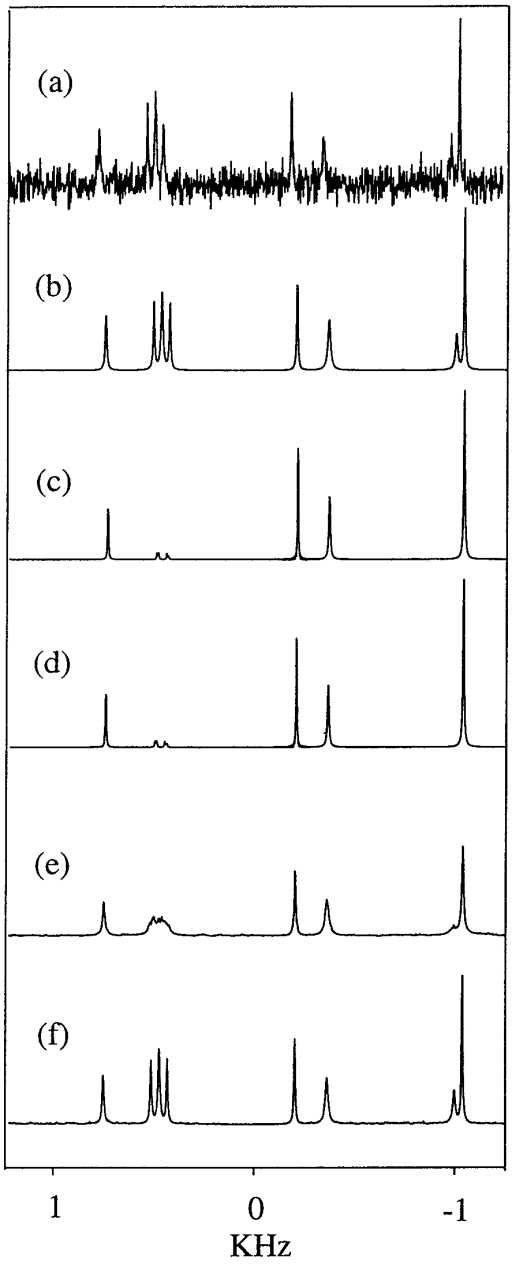

FIG. 1. (a)–(b) The 1D spectrum obtained by Fourier transform of a simulated FID containing 1024 complex data points with and without 10 dB additive noise. Parts (c)–(f) show the spectra calculated from the LP, MLP, TLS, and IQML algorithms by averaging the 50 spectra calculated from the parameters estimated from each of the 50 simulated FIDs which contains a different random noise generated at a SNR of 10 dB. LP order and rank used for the calculation were 24.

is much more robust than TLS and LP methods over a broad range of conditions. The doublet around 1000 Hz cannot be fully resolved by these parametric methods because the number of the FID used for calculating the spectra shown in Figs. 2B and 2C is not large enough. The presence of noise-like peaks in Figs. 1 and 2 for the TLS and IQML is due to inaccurate estimation of resonances in a few of the calculations with different noise entries. Whereas in the LP calculation, these erroneous signals and very often the true signals cannot be selected according to our selection rules described in the text resulting in relatively smooth baselines.

CONCLUSION

Through the detailed study described above, we can conclude that the IQML method is the most accurate parametric method for estimating spectral parameters from simulated

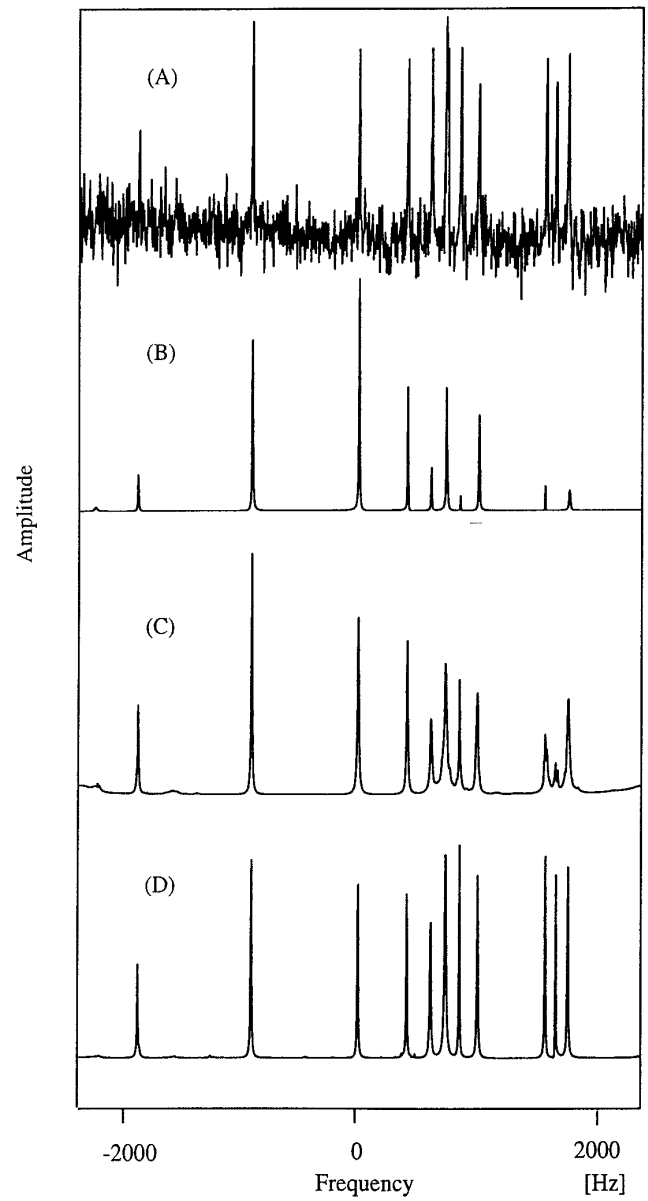

FIG. 2. (A) The 1D Fourier-transformed ^{13}C sucrose NMR spectrum. (B)–(D) The spectra obtained from the LP, TLS, and IQML methods, averaged over spectra calculated using 128 complex data points starting from the third, fourth, fifth, and sixth data points with the LP order being the even numbers between 12 and 40, and the rank used being 12.

and experimental FID data sets. However, due to the greater computational complexity of the IQML method, it is about a few times slower than the LP and TLS methods. In this study we have demonstrated that the IQML method is a powerful technique for direct quantitative spectral analysis in NMR spectroscopy, for instance, the quantitative analysis of ^{31}P NMR spectra *in vivo*. The IQML method can also be used to provide an excellent starting point for further spectral refinement after estimation of spectral parameters from the experimental data.

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REFERENCES

1. S. F. Gull, and D. J. Daniell, *Nature* **272,** 686–690 (1978).
2. E. D. Laue, J. Skilling, J. Staunton, S. Sibisi, and R. G. Brereton, *J. Magn. Reson.* **62,** 437–452 (1985).
3. G. Zhu, *J. Magn. Reson. B* **113,** 248–251 (1996).
4. H. Barkhuijsen, R. De Beer, W. M. M. J. Bovee, and D. Van Ormondt, *J. Magn. Reson.* **62,** 465–481 (1985).
5. R. Kumaresan, L. L. Scharf, and A. K. Shaw, *IEEE Trans. ASSP* **34,** 637–640 (1986).
6. G. Zhu, and A. Bax, *J. Magn. Reson.* **100,** 202 (1992).
7. C. F. Tirendi and J. F. Martin, *J. Magn. Reson.* **85,** 162–169 (1989).
8. G. W. Jeong, P. N. Borer, S. S. Wang, and G. C. Levy, *J. Magn. Reson. A* **103,** 123–134 (1994).
9. Y. Bresler and A. Macovski, *IEEE Trans. ASSP* **34,** 1081–1089 (1986).
10. Y. Hua, *IEEE Trans. Signal Process.* **42,** 2203–2204 (1994).
11. G. Zhu and Y. Hua, *Chem. Phys. Lett.* **264,** 424–428 (1997).